# QCD-DE: QCD topological vacuum as dark energy

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**Preamble**

One of the challenges of this lecture is that it lies at the crossroads of two fields: cosmology and particle physics. An additional difficulty arises in the strongly coupled, non-perturbative regime of QCD, where analytical calculations are often impossible. Consequently, analogies, simplified models, and exactly solvable theories in lower dimensions play an important role in developing physical intuition. However, these arguments must be distinguished from rigorous derivations in four-dimensional QCD.

Since this lecture addresses graduate students from different backgrounds, we will begin by introducing the basic concepts of dark energy for those unfamiliar with cosmology, followed by an introduction to QCD for those without formal training in particle physics, before exploring the possible connections between them.

# 1 Introduction

Could the fundamental nature of dark energy (DE) be rooted in non-perturbative QCD effects? Urban and Zhitnitsky (2009, 2010b,a) propose that the origin of DE may reside in the topological structure of the QCD vacuum. This idea is further developed in Zhitnitsky (2013), where it is argued that the vacuum energy responsible for cosmic acceleration originates from tunnelling between distinct topological sectors of QCD in an expanding Universe[1].

This idea has recently been implemented within a Big Bang cosmological framework (Van Waerbeke and Zhitnitsky, 2025). In this scenario, the DE contribution to the Friedmann equation is not strictly constant, but instead scales with the Hubble parameter, thereby offering an explanation for why DE becomes cosmologically relevant only at late times. The mechanism relies exclusively on non-perturbative QCD dynamics in expanding spacetime and does not introduce any additional fundamental fields. *It therefore constitutes a topological, rather than dynamical, explanation of the origin of dark energy.*

This lecture is organised as follows. Section 2 reviews the standard cosmological picture of dark energy, from ΛCDM to evolving equations of state and the three fundamental challenges associated with dark energy. Section 3 introduces the QCD vacuum and explains how its non-trivial topological structure may generate macroscopic effects. Section 4 develops the proposed connection between QCD vacuum topology and dark energy, from Zel'dovich's conjecture to simplified theoretical models. Section 5 extends the framework to a time-dependent Universe, introducing the adiabatic condition and the phenomenological activation of dark energy. Section 6 confronts the resulting cosmological model with recent observations, while Section 7 revisits the three fundamental challenges of dark energy and assesses how QCD-DE might address them.

# 2 The standard picture of dark energy

**Preamble: Natural units and conventions**

In particle physics, natural units are commonly used by setting the speed of light and the reduced Planck constant to unity ($c = \hbar = 1$). Consequently, energy, mass, temperature, inverse length, and inverse time can all be expressed in energy units, typically electronvolts (eV). Temperature is converted into energy by additionally setting Boltzmann's constant $k_B = 1$: for example, $T = 1\,\mathrm{MeV}$ corresponds to approximately $1.16 \times 10^{10}\,\mathrm{K}$. Similarly, a length of 1 fm ($10^{-15}\,\mathrm{m}$) corresponds to approximately $5.07\,\mathrm{GeV}^{-1}$. The Hubble constant, which has dimensions of inverse time, becomes:

$$H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \simeq 1.49 \times 10^{-33}\,\mathrm{eV}.$$

[1] See (Zhitnitsky, 2015; Van Waerbeke and Zhitnitsky, 2025) for references and a brief overview of this idea.

Natural units are convenient because the characteristic energy scale $\Lambda_{\rm QCD} \approx 100$–$200\,\text{MeV}$ simultaneously determines the typical masses, distances, and temperatures associated with strong-interaction physics.

## 2.1 A brief history of the Universe

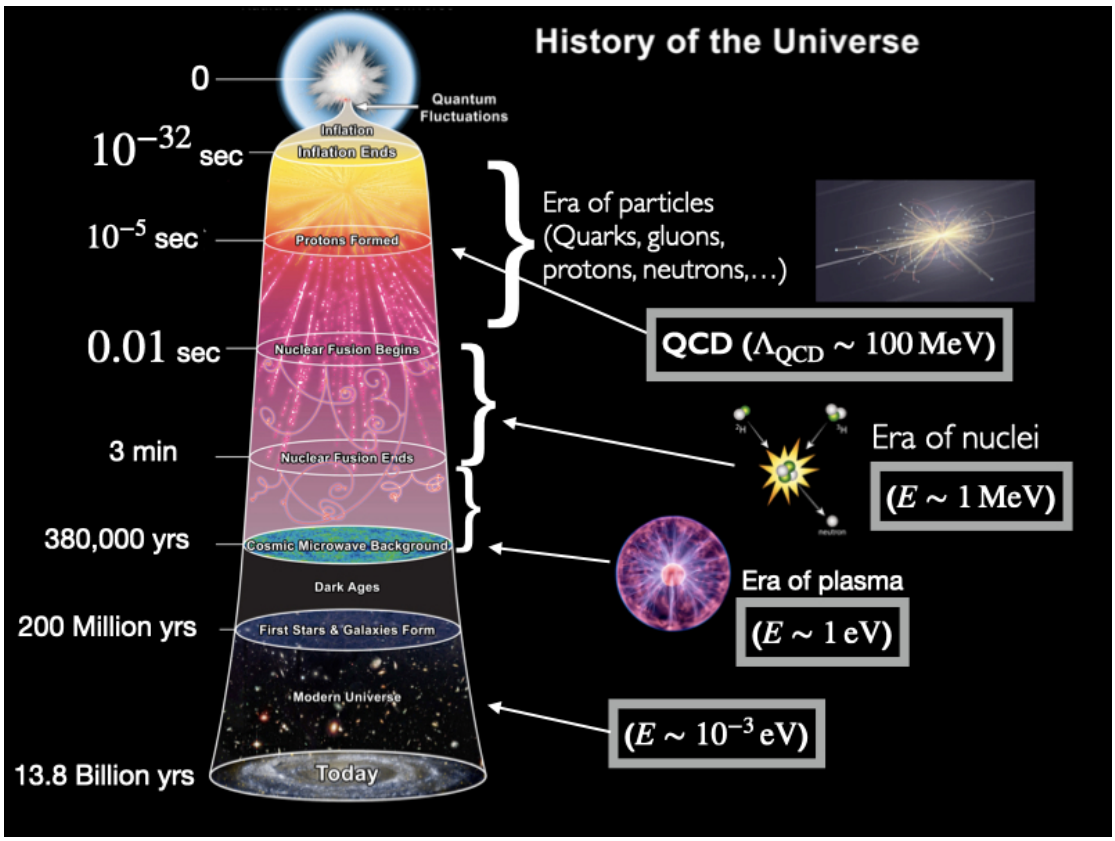



Figure 1: History of the Universe with temperatures in natural units. Image credit: adapted from BICEP2 Collaboration / CERN / NASA.

The Universe began in an extremely hot and dense state. Following inflation, which ended around $10^{-32}\,\text{s}$, the Universe entered a radiation-dominated era, filled with a thermal plasma of elementary particles. As expansion proceeded, the temperature decreased, successively crossing the characteristic energy scales of fundamental physics.

A particularly important milestone occurred around $t \sim 10^{-5}\,\text{s}$, when the temperature dropped to $T \sim \Lambda_{\rm QCD} \sim 100$–$200\,\text{MeV}$. Quarks and gluons, previously forming a deconfined plasma, became confined into hadrons, including protons and neutrons. This QCD crossover established the strongly coupled, non-perturbative regime characterized by quark and gluon condensates and a non-trivial topological vacuum structure. As we shall explore, these properties may have consequences extending to cosmological scales.

When the temperature reached $T \sim 1\,\text{MeV}$, primordial nucleosynthesis began, producing light nuclei. Approximately 380,000 years later, at $T \sim 0.3\,\text{eV}$, electrons combined with nuclei to form neutral atoms, releasing the cosmic microwave background (CMB). The subsequent formation of stars and galaxies marked the emergence of the large-scale structures observed today.

A second crucial epoch occurred around redshift $z \sim 2$, approximately 10 billion years ago, when dark energy may have begun to emerge dynamically. Although cosmic expansion had been proceeding since the earliest epochs, its accelerating phase began only around $z \sim 0.7$, approximately 6 billion years ago. Understanding why a vacuum contribution associated with QCD

physics becomes cosmologically relevant so late is a central motivation for the model developed in this lecture.

## 2.2 ΛCDM: The standard model of cosmology and its limitations

The ΛCDM model is currently the standard cosmological framework, providing a remarkably successful description of the Universe from the CMB to the present epoch. It combines cold dark matter (CDM), responsible for the gravitational growth of cosmic structures, with a cosmological constant Λ, which drives the observed accelerated expansion. Within this framework, dark energy is identified with a constant vacuum energy density, whose measured value is approximately

$$\rho_{\rm DE} = \rho_\Lambda \simeq (2.3\,{\rm meV})^4 = \text{constant}. \tag{1}$$

The expansion history of the Universe is governed by the Friedmann equation, derived from Einstein's field equations. For a spatially flat Universe, it can be written as

$$H^2(a) = \frac{8\pi G}{3}\left[\rho_{r,0}a^{-4} + \rho_{m,0}a^{-3} + \rho_\Lambda\right], \tag{2}$$

where $H(a) = \dot{a}/a$ is the Hubble expansion rate, $a$ is the scale factor normalized to unity today, and $\rho_{r,0}$ and $\rho_{m,0}$ are the present-day radiation and matter energy densities, respectively. The matter component includes both baryons and cold dark matter. Radiation decreases as $a^{-4}$, matter as $a^{-3}$, while the cosmological constant remains unchanged. Consequently, the Universe evolves from radiation domination to matter domination, and eventually to DE domination.

The cosmological constant was originally introduced by Einstein as an additional term in his gravitational field equations. It can equivalently be interpreted as a contribution to the energy-momentum tensor, corresponding to a perfect fluid with negative pressure.

Unlike matter and radiation, whose energy densities decrease as the Universe expands, the density associated with Λ remains constant. Consequently, its relative contribution grows with cosmic time, eventually dominating the expansion and producing cosmic acceleration.

The success of ΛCDM rests on its ability to describe a broad range of independent observations using only six fundamental cosmological parameters. These include CMB temperature and polarization anisotropies, baryon acoustic oscillations, Type Ia supernova distances, and the large-scale distribution of galaxies. Despite some observational tensions, ΛCDM remains the reference model against which alternative DE theories are evaluated.

However, its empirical success does not explain the physical origin of the cosmological constant. In particular, why should the vacuum energy density be so extraordinarily small compared with the characteristic energy scales of particle physics? Furthermore, why does DE become cosmologically important only in the relatively recent Universe? These unresolved questions motivate the search for a microscopic explanation of dark energy, potentially rooted in known fundamental interactions rather than additional hypothetical fields.

## 2.3 The dark energy equation of state

In cosmology, the equation of state (EoS) describes the relationship between the pressure $P$ and energy density $\rho$ of a component of the Universe. It is conventionally characterized by the dimensionless parameter

$$w \equiv \frac{P}{\rho}. \tag{3}$$

The equation of state determines how the energy density evolves as the Universe expands. For a separately conserved component, energy conservation implies

$$\dot{\rho} + 3H(\rho + P) = 0, \tag{4}$$

which, for constant $w$, gives $\rho \propto a^{-3(1+w)}$. Non-relativistic matter has $w = 0$, radiation has $w = 1/3$, while a cosmological constant has $w = -1$, corresponding to a constant energy density.

The equation of state also determines whether a component contributes to cosmic acceleration. From the acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P), \tag{5}$$

a dominant component produces accelerated expansion when $w < -1/3$.

For dark energy, measuring $w_{\rm DE}(z)$ is a powerful way to distinguish between competing cosmological models. A constant $w_{\rm DE} = -1$ is consistent with ΛCDM, whereas deviations or time evolution could indicate new physics.

The equation of state can also be reconstructed from the expansion history, allowing observations to constrain dark energy without specifying its microscopic origin. In QCD-DE, this distinction is essential: $w_{\rm DE}(z)$ is an effective quantity derived from the expansion dynamics, rather than the equation of state of a new fundamental field.

### 2.4 The CPL parametrization: evolving dark energy

The Chevallier–Polarski–Linder (CPL) parametrization is widely used to investigate whether dark energy evolves with cosmic time. Unlike ΛCDM, which assumes a constant vacuum energy density, CPL allows the DE equation of state to vary with the scale factor $a = 1/(1+z)$, according to

$$w_{\rm DE}(a) = w_0 + w_a(1-a), \tag{6}$$

where $w_0$ is the present-day equation-of-state parameter and $w_a$ describes its evolution. Equivalently,

$$w_{\rm DE}(z) = w_0 + w_a\frac{z}{1+z}. \tag{7}$$

The ΛCDM model corresponds to the special case $w_0 = -1$ and $w_a = 0$. Assuming that dark energy is separately conserved, its density evolves as

$$\rho_{\rm DE}(a) = \rho_{\rm DE,0}\, a^{-3(1+w_0+w_a)} \exp\left[3w_a(a-1)\right]. \tag{8}$$

The CPL parametrization introduces only two additional parameters relative to ΛCDM, while capturing a broad range of possible DE histories. It provides a convenient framework for combining cosmological observations and testing departures from a cosmological constant.

Recent results from the Dark Energy Spectroscopic Instrument (DESI), when combined with CMB and Type Ia supernova measurements, suggest a preference for evolving dark energy over ΛCDM, with statistical significances depending on the datasets considered. Some preferred CPL solutions exhibit a crossing of the so-called phantom divide, $w_{\rm DE} = -1$, implying that the effective equation of state evolves between the quintessence regime ($w_{\rm DE} > -1$) and the phantom regime ($w_{\rm DE} < -1$).

*CPL is a purely phenomenological description: it parametrizes the evolution of DE without identifying its physical origin or underlying microscopic mechanism.* Moreover, crossing the phantom divide is theoretically non-trivial for conventional scalar-field models and may require additional degrees of freedom or unconventional dynamics. These limitations motivate the search for physically grounded alternatives, such as QCD vacuum energy, capable of generating an evolving effective equation of state without introducing new fundamental fields.

## 2.5 Why is phantom crossing theoretically non-trivial?

Crossing the phantom divide ($w_{\rm DE} = -1$) within a consistent physical theory is surprisingly difficult. Consider a minimally coupled canonical scalar field $\phi$, commonly invoked to describe dynamical dark energy. Its energy density and pressure are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{9}$$

where $V(\phi)$ is the scalar potential. Consequently,

$$w_\phi = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}. \tag{10}$$

For positive energy density, the canonical kinetic term ensures $w_\phi \geq -1$. Reaching the phantom regime would require reversing the sign of the kinetic term, producing a ghost field with negative kinetic energy. Such fields generally suffer from severe instabilities, including an unbounded Hamiltonian and potentially catastrophic vacuum decay.

Crossing $w = -1$ is therefore impossible for a single, minimally coupled scalar field with a conventional kinetic term. More elaborate constructions, involving multiple fields, non-canonical kinetic interactions, or modified gravity, may permit phantom crossing, although their theoretical consistency requires careful examination.

Phantom crossing does not necessarily imply the existence of a fundamental phantom field. An effective equation of state reconstructed from the cosmological expansion history may cross $w = -1$ without introducing negative-energy degrees of freedom. This distinction is particularly relevant for QCD-DE, where dark energy originates from the non-perturbative QCD vacuum rather than a dynamical scalar field. *The possibility of reproducing phantom crossing without fundamental ghosts constitutes an important motivation for investigating this alternative framework.*

## 2.6 The three fundamental challenges for DE

### 2.6.1 Explaining the small amplitude of dark energy

The first fundamental challenge for any DE theory is to explain why the observed energy density is extraordinarily small compared with the characteristic scales of particle physics. Cosmological observations give

$$\rho_{\rm DE}^{\rm obs} \simeq (2.3 \times 10^{-3}\,{\rm eV})^4 \simeq 2.8 \times 10^{-11}\,{\rm eV}^4. \tag{11}$$

### 2.6.2 Why does DE become important only at late times?

The second fundamental challenge is to explain why dark energy becomes cosmologically important only in the relatively recent Universe, at redshifts $z \lesssim 2$.

In the standard ΛCDM model, the DE density remains constant, while matter dilutes as $a^{-3}$. Their ratio therefore evolves according to

$$\frac{\rho_{\rm DE}(z)}{\rho_m(z)} = \frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}(1+z)^{-3}. \tag{12}$$

Consequently, DE becomes increasingly important as the Universe expands. For typical cosmological parameters, matter and dark energy reach equal densities near $z \simeq 0.3$, while accelerated expansion begins around $z \simeq 0.7$.

Although this evolution is mathematically straightforward, it does not explain why the present-day densities of matter and dark energy are comparable. This is the *cosmological coincidence problem*: why do we happen to observe the Universe during this relatively brief transition between matter domination and DE domination?

For QCD-based dark energy, the question is even more specific. If its microscopic origin was established in the early Universe, why would its gravitational contribution become significant only billions of years later?

A successful theory should explain whether this late-time behaviour results simply from the dilution of matter or from an additional physical mechanism that activates the vacuum contribution as the expansion rate decreases. *In QCD-DE, the proposed mechanism is the connection with the adiabatic evolution of topological vacuum configurations.*

### 2.6.3 Does dark energy evolve with time?

The third challenge concerns the possible time evolution of dark energy. In the standard ΛCDM model, the DE density is strictly constant. Any statistically significant departure from this behaviour would have profound implications for our understanding of the quantum vacuum and the origin of cosmic acceleration.

Recent results from the Dark Energy Spectroscopic Instrument (DESI) have renewed interest in this possibility. When baryon acoustic oscillation measurements are combined with CMB and Type Ia supernova data, some analyses favour an evolving equation of state over a cosmological constant, with reported significances reaching approximately 2.8–4.2$\sigma$, depending on the dataset combination. However, these results remain controversial and require independent confirmation. The statistical preference depends on the supernova compilation, the adopted parametrization of dark energy, and the treatment of observational systematics. Moreover, the apparent significance is obtained from combined datasets rather than from DESI measurements alone. It would therefore be premature to interpret these findings as evidence for a discovery of evolving dark energy.

Nevertheless, DESI is not the only motivation for investigating this possibility. Independent constraints from the Dark Energy Survey (DES), different supernova compilations, CMB measurements, and weak gravitational lensing provide complementary tests of the expansion and growth histories. Although these observations do not independently establish evolving dark energy, some combinations exhibit tensions with the simplest ΛCDM description.

The persistent discrepancy between local measurements of the Hubble constant and values inferred from the early Universe provides an additional motivation for exploring alternatives to ΛCDM. However, late-time DE evolution alone does not generally resolve this tension.

Some preferred phenomenological solutions suggest that $w_{\rm DE}(z)$ may cross the phantom divide $w=-1$, a behaviour that is difficult to reproduce with conventional canonical scalar fields.

Ultimately, demonstrating that dark energy evolves would be only the first step. A successful theory must also explain the physical mechanism responsible for this evolution, rather than merely introducing additional parameters to reproduce the observations.

# 3 QCD vacuum energy

## 3.1 Quantum chromodynamics: the theory of the strong interaction

Quantum chromoDynamics (QCD) is the quantum field theory describing the strong interaction. It governs the interactions between quarks and gluons, the elementary constituents responsible for the structure of protons, neutrons, and other hadrons. QCD is an essential component of the Standard Model of particle physics.

The theory emerged in the early 1970s to resolve several puzzles concerning the strong interaction. QCD provided a unified explanation by describing quarks as particles carrying one of three colour charges, interacting through the exchange of gluons. Mathematically, it is a non-Abelian [2] gauge theory based on the symmetry group $SU(3)_{\text{colour}}$. Unlike photons in electromagnetism, gluons themselves carry colour charge and therefore interact with one another. This self-interaction produces two remarkable properties.

The first is asymptotic freedom, discovered by Gross, Wilczek, and Politzer in 1973: the strong coupling becomes weaker at high energies, allowing quarks to behave approximately as free particles in sufficiently energetic collisions. The second is confinement: at low energies, quarks and gluons are never observed as isolated particles but remain bound inside colour-neutral hadrons.

Over the past five decades, QCD has achieved extraordinary experimental success. Perturbative calculations accurately describe high-energy scattering, jet production, and numerous processes measured at particle accelerators, including the Large Hadron Collider. At low energies, where perturbation theory fails, lattice QCD provides first-principles numerical predictions for hadron masses, decay constants, and other observables, often agreeing with experiments at percent-level precision or better.

### 3.2 QCD vacuum energy: local fluctuations and global topology

In classical physics, a vacuum is simply empty space. Quantum physics gives a radically different picture: even in the absence of particles, quantum fields remain present and exhibit fluctuations. The vacuum is therefore not truly empty but corresponds to the lowest-energy state of interacting quantum fields. These fluctuations can contribute to the vacuum energy, with measurable consequences such as the Casimir effect (Plunien et al., 1986).

In Quantum chromodynamics, the vacuum has a particularly rich structure. Strong interactions between quarks and gluons generate non-perturbative condensates, characterized by the QCD energy scale $\Lambda_{\text{QCD}} \sim 100$–$200\,\text{MeV}$. The characteristic vacuum energy density is consequently of order

$$|\rho_{\text{vac}}^{\text{QCD}}| \sim \Lambda_{\text{QCD}}^4. \tag{13}$$

An important distinction must be made between two contributions to QCD vacuum physics.

- **Local fluctuations.** The first contribution is associated with conventional, local fluctuations of physical degrees of freedom. Because QCD possesses a mass gap [3], correlations between gauge-invariant local operators generally decay exponentially at large separations,

$$C(r) \sim \exp(-mr), \tag{14}$$

  where $m$ represents the mass of the lightest state coupled to the operators. These effects are therefore short-ranged and cannot directly communicate information across cosmological distances.

[2] QCD is a gauge theory based on the symmetry group $SU(3)$. Quarks carry colour charge and interact through gluons, which themselves carry colour charge. Unlike photons in electromagnetism, gluons interact directly with one another. These self-interactions are responsible for much of the rich non-linear behaviour of QCD, including asymptotic freedom at high energies and the emergence of strongly coupled dynamics at low energies.

[3] A mass gap means that the lowest physical excitation above the vacuum has a strictly positive mass. Consequently, correlations between gauge-invariant local operators generally decay exponentially at large separations, with a characteristic length determined by the inverse mass of the lightest state coupled to those operators. This behaviour normally prevents local fluctuations from producing long-range effects. The possibility that topological vacuum contributions remain sensitive to much larger distances is therefore particularly remarkable.

- **Topological contributions.** The second contribution is associated with the non-trivial topology of the QCD gauge vacuum. Distinct topological sectors, connected through quantum tunnelling [4], contribute to the vacuum energy. Their effects are non-dispersive: they cannot be represented solely through propagating physical particles. In particular, the topological susceptibility [5] contains a contact contribution that is not governed by conventional massive-particle propagation.

This structure motivates the possibility that QCD vacuum energy exhibits sensitivity to global geometry and boundary conditions, even on scales much larger than $\Lambda_{\rm QCD}^{-1}$. Such long-range sensitivity does not imply a new propagating massless particle. Whether it produces a cosmologically significant contribution in four-dimensional expanding spacetime remains an open theoretical question, central to the QCD-DE proposal.

## 3.3 Long-range quantum effects in gauge theories

Quantum gauge theories can exhibit physical effects over distances much larger than the characteristic scales of their local interactions. Such effects arise from the global properties of gauge fields and their topology. This possibility may seem counterintuitive, so to illustrate this property, we will examine two well-established examples from quantum physics: the Aharonov–Bohm effect and the Aharonov–Casher effect. Both demonstrate how global gauge properties can influence measurable quantities even in regions where the corresponding local fields vanish. They provide useful physical analogies for understanding how the non-trivial topology of the QCD vacuum might generate sensitivity to cosmological scales.

### 3.3.1 The Aharonov–Bohm effect: a long-range quantum phenomenon

The Aharonov–Bohm effect, predicted in 1959 (Aharonov and Bohm, 1959), provides a striking demonstration that electromagnetic potentials can influence quantum particles even where the local electromagnetic fields vanish. Consider an electron beam split into two paths passing on opposite sides of a long solenoid containing a confined magnetic field. Outside the solenoid, the magnetic field vanishes, yet the electrons acquire different quantum phases, producing a measurable shift in their interference pattern.

The phase difference is

$$\Delta\varphi = \frac{e}{\hbar}\oint \mathbf{A}\cdot d\mathbf{l} = \frac{e\Phi_B}{\hbar}, \tag{15}$$

where $\mathbf{A}$ is the vector potential and $\Phi_B$ is the enclosed magnetic flux.

This phase shift depends on the total enclosed flux, not on the distance between the electrons and the solenoid. The effect therefore demonstrates a form of non-local sensitivity to global gauge configurations, without requiring a long-range magnetic field.

[4] In QCD, gauge-field configurations can belong to distinct topological sectors, labelled by an integer winding number. These sectors are separated by energy barriers that cannot be crossed through small classical fluctuations. Quantum mechanics nevertheless permits transitions between them by tunnelling. In the Euclidean formulation, such transitions are described semiclassically by instantons, finite-action solutions of the Yang–Mills equations carrying nonzero topological charge. Their contributions to physical observables are non-perturbative, typically involving factors proportional to $\exp(-8\pi^2/g^2)$, which cannot be obtained from a conventional power-series expansion in the coupling $g$. Tunnelling between sectors contributes to the structure of the QCD vacuum, including its dependence on the $\theta$ parameter and its topological susceptibility.

[5] The topological susceptibility measures the response of the vacuum energy to the QCD $\theta$ parameter, or equivalently the fluctuations of the total topological charge. It can be expressed as an integral of the correlation function of the topological charge density. Importantly, it contains a non-dispersive contact contribution that cannot be reconstructed solely from propagating physical states. This property is central to the proposed connection between QCD vacuum topology and cosmological dark energy.

The phenomenon has been experimentally confirmed repeatedly, notably through electron interferometry and the landmark experiments of Tonomura et al. (1986), which used magnetically shielded toroidal magnets.

### 3.3.2 The Aharonov–Casher effect: long-range sensitivity to electric charge

The Aharonov–Casher effect, predicted in 1984 (Aharonov and Casher, 1984), is the electromagnetic counterpart of the Aharonov–Bohm effect. It demonstrates that a neutral particle carrying a magnetic moment can acquire a measurable quantum phase while moving around an electrically charged object, even without experiencing a classical force. It was experimentally confirmed in 1989 through neutron interferometry (Cimmino et al., 1989).

Consider a neutron travelling along two paths that enclose a long, electrically charged wire. The neutron has no electric charge but possesses a magnetic dipole moment. Its interaction with the electric field produces a relative quantum phase between the two paths, given in the idealized geometry by

$$\Delta\varphi_{\mathrm{AC}} = \frac{\mu\lambda}{\hbar\epsilon_0 c^2}, \tag{16}$$

where $\mu$ is the neutron magnetic moment and $\lambda$ is the enclosed electric charge per unit length.

This phase depends on the enclosed charge rather than the distance from the wire. It illustrates how quantum interference can retain sensitivity to global configurations, even when local forces vanish.

## 3.4 Can QCD vacuum energy explain the observed dark energy?

Having established that the QCD vacuum possesses a non-trivial energy density, we can ask whether it could account for the dark energy observed in the Universe. At first sight, the answer is clearly no: the characteristic QCD vacuum energy is enormously larger than the cosmological value.

The natural energy scale of the QCD vacuum is set by $\Lambda_{\mathrm{QCD}} \sim 100$–$200\,\mathrm{MeV}$, suggesting a vacuum energy density of order

$$|\rho_{\mathrm{vac}}^{\mathrm{QCD}}| \sim \Lambda_{\mathrm{QCD}}^4 \sim 10^{32}\text{–}10^{33}\,\mathrm{eV}^4. \tag{17}$$

By comparison, cosmological observations indicate a dark-energy density of approximately

$$\rho_{\mathrm{DE}}^{\mathrm{obs}} \simeq (2.3\times10^{-3}\,\mathrm{eV})^4 \simeq 2.8\times10^{-11}\,\mathrm{eV}^4. \tag{18}$$

The discrepancy is therefore staggering:

$$\frac{|\rho_{\mathrm{vac}}^{\mathrm{QCD}}|}{\rho_{\mathrm{DE}}^{\mathrm{obs}}} \sim 10^{43}\text{–}10^{44}. \tag{19}$$

If the full QCD vacuum energy gravitated as a cosmological constant, its contribution would overwhelm the observed cosmic expansion. Furthermore, the conventional QCD vacuum energy is negative, whereas the observed DE density is positive.

This enormous mismatch illustrates the cosmological constant problem: why does the gravitational effect of vacuum energy differ so dramatically from the characteristic scales of quantum field theory? *The crucial question is therefore not whether QCD possesses sufficient vacuum energy, but whether a physical mechanism can isolate a tiny, positive gravitational contribution from it.* An important historical step toward this idea was proposed by Yakov Zel'dovich in 1967, who suggested that gravitational interactions between quantum vacuum fluctuations might generate a much smaller cosmological energy scale.

# 4 From Zel'dovich's conjecture to QCD dark energy

## 4.1 Cosmological constant or vacuum energy?

As first explicitly recognised by Lemaître in 1934, General Relativity cannot distinguish between a cosmological constant introduced as a geometrical term and a constant vacuum energy contributing to the energy-momentum tensor.

Einstein's field equations, including the cosmological constant $\Lambda$, are written as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{20}$$

Lorentz invariance requires the vacuum expectation value of the energy-momentum tensor to take the form

$$\langle T_{\mu\nu} \rangle_{\rm vac} = -\rho_{\rm vac} g_{\mu\nu}. \tag{21}$$

Moving the cosmological constant to the right-hand side of Einstein's equations shows that the two descriptions are mathematically equivalent, with

$$\rho_{\rm vac} = \frac{\Lambda}{8\pi G}. \tag{22}$$

Consequently, observations of cosmic expansion alone cannot determine whether a constant DE contribution originates from a geometrical cosmological constant or from the quantum vacuum.

This equivalence is fundamental to the cosmological constant problem. It also provides the historical context for Zel'dovich's 1967 conjecture: rather than treating $\Lambda$ as an unexplained geometrical parameter, one might attempt to derive its magnitude from the microscopic physics of quantum vacuum fluctuations and their gravitational interactions.

## 4.2 Zel'dovich's 1967 conjecture

In 1967, Yakov Zel'dovich proposed a remarkable idea: the cosmological constant might originate from the gravitational interactions of quantum vacuum fluctuations. Rather than identifying the observed vacuum energy with the enormous zero-point energy predicted by quantum field theory, he suggested that gravitational corrections could generate a much smaller residual contribution.

His dimensional argument considers virtual particles characterized by a mass scale $m$, separated by their Compton wavelength $\lambda \sim 1/m$. Their characteristic gravitational interaction energy is $E_{\rm grav} \sim Gm^2/\lambda$. Dividing this energy by a characteristic volume $\lambda^3$ gives

$$\rho_{\rm vac} \sim Gm^6 = \frac{m^6}{M_{\rm Pl}^2}, \tag{23}$$

where $M_{\rm Pl} = G^{-1/2}$ is the Planck mass.

Taking $m$ near the hadronic mass scale produces an estimate dramatically smaller than the usual quartic vacuum-energy contribution, $\rho_{\rm vac} \sim m^4$.

Although this conjecture does not constitute a microscopic derivation, nor does it establish the required sign of the vacuum pressure, it introduced an important possibility: *the cosmological constant might reflect a residual quantum effect involving particle physics and gravity.*

Zel'dovich's estimate dramatically reduces the discrepancy between the characteristic vacuum energy of particle physics and the observed DE density, from more than 40 orders of magnitude to approximately six orders of magnitude when the proton mass is used as the characteristic scale. Although still about a million times larger than the observed value, this represents a remarkable improvement obtained from a simple dimensional argument.

### 4.3 From Minkowski subtraction to topological dark energy

Building on Zel'dovich's (1967) conjecture, and its subsequent formalisation by Urban and Zhitnitsky (2010b), A. Zhitnitsky proposed that the gravitationally relevant vacuum energy should be defined relative to Minkowski spacetime. In this framework, the vacuum energy present in flat spacetime is taken as the zero-point energy, while the cosmologically relevant energy density is associated with the change induced by the expanding background.

Hence, the central quantity is not the absolute vacuum energy, but the difference between its value in an expanding Universe and in Minkowski spacetime [6]:

$$\rho_{\rm DE} = E_{\rm vac}^{\rm FLRW} - E_{\rm vac}^{\rm Minkowski}. \tag{24}$$

Here FLRW denotes the Friedmann–Lemaître–Robertson–Walker cosmological background, whose expansion is characterised by the Hubble parameter $H(t)$. The residual vacuum energy may depend on $H$, its time derivatives, and other geometric properties of spacetime.

The QCD vacuum energy in Minkowski spacetime is negative and of order the fourth power of the confinement scale (Chodos et al., 1974b,a):

$$E_{\rm vac}^{\rm Minkowski} = -\Lambda_{\rm QCD}^4. \tag{25}$$

*The essential question is therefore how this vacuum energy changes when the background spacetime expands.*

**Local curvature corrections versus topological contributions.** For conventional quantum fields described by a local, generally covariant effective action, the leading geometric corrections involve curvature invariants. In a spatially flat FLRW spacetime, the Ricci scalar is

$$R = 6(2H^2 + \dot{H}). \tag{26}$$

Consequently, the leading local curvature-dependent contribution to the vacuum energy is expected to scale schematically as (Shapiro and Solà, 2000, 2002)

$$\Delta E_{\rm vac}^{\rm local} \sim H^2 \Lambda_{\rm QCD}^2 \sim 10^{-52}\,{\rm eV}^4. \tag{27}$$

This contribution depends quadratically, rather than linearly, on the expansion rate. It is far too small to account for the observed DE density. However, this reasoning assumes that the vacuum response is only represented by local operators in a derivative expansion. It does not constrain contributions associated with the global topological structure of the QCD vacuum.

As discussed in Section 3, QCD gauge fields possess distinct topological sectors connected through non-perturbative tunnelling processes. These sectors contribute to the vacuum energy without corresponding to conventional propagating particles. Their contribution is non-dispersive: it cannot be reconstructed from the spectrum of physical particle excitations, and it is not associated with the propagation of a massive physical state.

Although QCD's massive correlations are exponentially suppressed at large distances, its topological vacuum structure may retain sensitivity to global properties of spacetime. The Aharonov–Bohm and Aharonov–Casher effects discussed earlier provide useful analogies for such non-local quantum sensitivity.

This motivates the conjecture that the topological contribution to the QCD vacuum need not follow the conventional local curvature expansion $\propto H^2$. In particular, Zhitnitsky proposed

[6] Minkowski spacetime corresponds to the flat, non-expanding reference background, with $H = 0$. The subtraction prescription is a defining assumption of the QCD-DE framework, rather than a general consequence of Einstein's equations.

that its leading response to cosmological expansion could be linear in the Hubble parameter (Zhitnitsky, 2015):

$$\Delta E_{\rm vac}^{\rm topo} \sim H\Lambda_{\rm QCD}^3. \tag{28}$$

Since $H \ll \Lambda_{\rm QCD}$, this contribution would be enormously larger than the conventional quadratic correction:

$$\frac{\Delta E_{\rm vac}^{\rm topo}}{\Delta E_{\rm vac}^{\rm local}} \sim \frac{H\Lambda_{\rm QCD}^3}{H^2\Lambda_{\rm QCD}^2} = \frac{\Lambda_{\rm QCD}}{H} \gg 1. \tag{29}$$

The physical interpretation is not that QCD introduces a new massless particle or a conventional long-range force. Rather, the topological vacuum energy could respond to the global geometry of the Universe through its non-trivial gauge fields topological structure.

**The de Sitter limit and the dark-energy amplitude.** A complete calculation of $E_{\rm vac}^{\rm FLRW}$ in a general, time-dependent FLRW background is not presently available. However, the de Sitter case, characterised by a constant Hubble parameter $H(t) = \bar{H}$, provides a particularly useful theoretical limit.

In the QCD-DE framework, Zhitnitsky (2015) calculated[7] the following leading expression for the vacuum energy:

$$E_{\rm vac}^{\rm FLRW} = -\Lambda_{\rm QCD}^4\left(1 - c_{\bar{H}}\frac{\bar{H}}{\Lambda_{\rm QCD}}\right). \tag{30}$$

The dimensionless coefficient $c_{\bar{H}}$ is estimated to be of order

$$c_{\bar{H}} \sim \frac{m_q}{\Lambda_{\rm QCD}} \sim 0.01 - 0.02,$$

where $m_q \sim m_u \sim m_d$ denotes a light-quark mass[8].

Substituting Eq. (30) and Eq. (25) into Eq. (24) gives a positive DE density:

$$\rho_{\rm DE} = c_{\bar{H}}\bar{H}\Lambda_{\rm QCD}^3. \tag{31}$$

The enormous Minkowski contribution proportional to $\Lambda_{\rm QCD}^4$ has cancelled, leaving a much smaller residual contribution controlled by the cosmological expansion rate.

In the pure de Sitter limit, the Friedmann equation becomes

$$\bar{H}^2 = \frac{8\pi G}{3}\rho_{\rm DE}. \tag{32}$$

Combining this relation with Eq. (31), and using natural units with $G = M_{\rm PL}^{-2}$, where $M_{\rm PL}$ denotes the unreduced Planck mass, yields

$$\bar{H} = c_{\bar{H}}\frac{8\pi\Lambda_{\rm QCD}^3}{3M_{\rm PL}^2}, \qquad \rho_{\rm DE} = c_{\bar{H}}^2\frac{8\pi\Lambda_{\rm QCD}^6}{3M_{\rm PL}^2}.$$

These expressions illustrate how the cosmological expansion scale can emerge from the QCD scale through gravitational suppression, without introducing an independent microscopic DE scale.

For $\Lambda_{\rm QCD} \sim 100\,\text{MeV}$ and $c_{\bar{H}} = 0.01 - 0.02$, the corresponding estimates are

[7] This calculation was done in the context of the toy-model presented in Section 4.5

[8] The ratio $m_q/\Lambda_{\rm QCD}$ is a QCD-related suppression factor associated with tunnelling contributions in the proposed framework.

$$\begin{cases} \bar{H} \simeq (1.7-3.4)\times 10^{-33}\,\mathrm{eV} \\ \quad\simeq 80-160\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \\ \rho_{\mathrm{DE}} \simeq (4-16)\times 10^{-11}\,\mathrm{eV}^4 \\ \quad\simeq (9-36)\times 10^{-30}\,\mathrm{g\,cm^{-3}}, \\ \bar{H}^{-1} \simeq 12-6\,\mathrm{Gyr}. \end{cases} \tag{33}$$

These values may be compared with the observed cosmological scales:

$$\begin{cases} H_0 \sim 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \\ \rho_{\mathrm{DE}}^{\mathrm{obs}} \sim 10^{-11}\,\mathrm{eV}^4, \\ t_0 \sim 14\,\mathrm{Gyr}. \end{cases}$$

The predicted scales are remarkably close to their observed counterparts, despite being determined by $\Lambda_{\mathrm{QCD}}$. This agreement is non-trivial: *the same mechanism that suppresses the enormous QCD vacuum energy also generates a cosmological expansion rate of approximately the observed magnitude.*

Nevertheless, these estimates describe the asymptotic de Sitter regime. In particular, $\bar{H}^{-1}$ is the characteristic de Sitter expansion timescale, not a calculation of the age of a Universe containing matter and radiation.

**The remaining theoretical challenge.** The existence and magnitude of a linear Hubble correction have not been established by a complete first-principles calculation in four-dimensional QCD on an expanding spacetime.

The proposal is motivated by the non-dispersive character of topological vacuum contributions and by results obtained in simpler gauge theories. Establishing whether the same mechanism operates quantitatively in cosmology remains a central theoretical challenge. This limitation motivates the following sections, where we explain why the full QCD calculation is presently inaccessible and how lower-dimensional toy models can provide insight into the proposed large-scale behaviour.

**A first-principles calculation in 4D remains inaccessible.** Although the arguments leading to Eq. (33) motivate a linear correction to the vacuum energy, one would ideally evaluate the vacuum energy in a four-dimensional FLRW geometry and subtract its Minkowski-space counterpart, thereby deriving the cosmologically relevant contribution without phenomenological assumptions.

Unfortunately, this calculation is currently beyond the reach of available analytical and numerical methods. Lattice QCD provides powerful non-perturbative calculations, but conventional simulations are formulated in Euclidean spacetime. A genuinely time-dependent FLRW background introduces additional difficulties, particularly when attempting to recover the real-time evolution of topological vacuum contributions. Furthermore, the proposed correction is extraordinarily small compared with the characteristic QCD vacuum energy, making its numerical extraction exceptionally demanding.

We therefore turn to two complementary toy models: the two-dimensional Schwinger model, which illustrates the sensitivity of topological vacuum contributions to large distances and time-dependent backgrounds, and a four-dimensional gauge theory in curved spacetime, which demonstrates how a correction linear in the curvature scale can emerge from non-trivial vacuum topology. These models retain essential features of the problem, such as distinct topological sectors, tunnelling, and non-dispersive vacuum contributions.

### 4.4 The two-dimensional toy model: topology and adiabatic evolution

The Schwinger model (Schwinger, 1962) is an exactly solvable quantum electrodynamics theory in two spacetime dimensions, widely used as a toy model of QCD (Coleman, 1976). Despite its simplicity, it exhibits several features reminiscent of QCD, including non-trivial topological sectors, a $\theta$-vacuum structure, and dynamical generation of a mass gap $m_\gamma$.

The model is formulated on a 2D Euclidean torus of characteristic size $L$. Ordinary massive excitations are expected to produce finite-size corrections exponentially suppressed as $\exp(-m_\gamma L)$. Remarkably, the topological contributions to the vacuum energy, chiral condensate, and topological susceptibility instead exhibit corrections proportional to $1/(m_\gamma L)$.

This power-law dependence originates from zero-mode determinants associated with instanton configurations. It demonstrates that topological vacuum observables can remain sensitive to macroscopic distances despite the presence of a mass gap.

The calculation can be extended to a time-dependent background by allowing the characteristic size of the system to evolve slowly,

$$L \longrightarrow L(t) = L + \dot{L}\Delta t. \tag{34}$$

The response of the topological vacuum to this evolution introduces an additional suppression relative to the static result. By identifying the characteristic inverse length scale with the asymptotic Hubble rate $\bar{H}$, and the mass gap $m_\gamma$ with $\Lambda_{\rm QCD}$, the corresponding QCD-DE contribution is motivated by the expression

$$\frac{\rho_{\rm DE}}{\Lambda_{\rm QCD}^4} \propto \frac{\bar{H}}{\Lambda_{\rm QCD}} \left(1 + \frac{|\dot{H}|}{H\bar{H}}\right)^{-1}. \tag{35}$$

The dimensionless quantity

$$\epsilon_{\rm ad} \equiv \frac{|\dot{H}|}{H\bar{H}} \tag{36}$$

characterizes the departure from adiabatic evolution. When $\epsilon_{\rm ad} \ll 1$, the topological vacuum can respond adiabatically to the cosmological expansion, and the DE contribution approaches its asymptotic de Sitter value. Conversely, when $\epsilon_{\rm ad} \gtrsim 1$, rapid cosmological evolution suppresses this contribution.

### 4.5 A four-dimensional model in curved spacetime

A second theoretical test of QCD-DE uses a four-dimensional Euclidean gauge theory on the curved manifold $\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}$. Here $\mathbb{H}^3_\kappa$ is a three-dimensional hyperbolic space with curvature scale $\kappa$, while $\mathbb{S}^1_{\kappa^{-1}}$ is a compact circle of circumference $\kappa^{-1}$. Unlike the Schwinger model, this construction retains the dimensionality and non-Abelian topological structure relevant to QCD.

The calculation exploits semiclassical gauge configurations carrying non-trivial holonomy, which characterises the gauge field around the compact direction. These configurations can be interpreted as monopole-instantons wrapping around the circle. Their tunnelling contributions are non-dispersive: they modify the vacuum energy without introducing propagating massless particles.

The principal result is that the vacuum-energy difference between curved and flat backgrounds contains a term linear in the curvature scale (Zhitnitsky, 2015):

$$\begin{aligned} \Delta E_{\rm vac} &\equiv E_{\rm vac}[\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}] - E_{\rm vac}[\mathbb{R}^3 \times \mathbb{S}^1] \\ &\simeq c_\kappa \kappa \Lambda^3_{\rm QCD}, \end{aligned} \tag{37}$$

where $c_\kappa$ is a dimensionless coefficient of order unity. Crucially, the correction is positive and linear in $\kappa$, rather than quadratic as expected from conventional local curvature expansions.

This result demonstrates how topological vacuum contributions can exhibit large-scale sensitivity despite a mass gap. However, the background is not de Sitter spacetime, and analogous holonomy-carrying solutions have not been constructed there. The cosmological extension remains conjectural: identifying $\kappa$ with the asymptotic Hubble rate $\bar{H}$ motivates

$$\rho_{\rm DE} \sim c_H \bar{H} \Lambda_{\rm QCD}^3. \tag{38}$$

Thus, the curved-space calculation supports the proposed linear scaling.

# 5 Time-dependent Hubble parameter

The QCD-DE mechanism discussed so far was formulated in the asymptotic de Sitter limit, where the Hubble parameter approaches a constant value, $H(t) = \bar{H}$. In this regime, the geometry is time-independent on the characteristic timescale of the expansion, allowing the topological vacuum configurations to be treated in a stationary background.

However, the actual Universe is not yet in this regime. Its expansion rate has evolved continuously through radiation domination, matter domination, and the recent onset of accelerated expansion. To describe QCD-DE at finite redshift, we must understand how the topological vacuum responds to a time-dependent Hubble parameter.

## 5.1 The adiabatic condition

The calculations motivating the topological vacuum contribution assume that the background changes sufficiently slowly for tunnelling processes between topological sectors to be described using approximately stationary configurations. This is the adiabatic approximation, which is implicit in calculations where parameters such as $\kappa$ in Eq. (37), or the Hubble parameter itself, are held constant.

The characteristic timescale over which the Hubble parameter changes is $t_H \sim H/|\dot{H}|$, while the expansion timescale is $H^{-1}$. Requiring the former to be much longer than the latter gives the **adiabatic condition**:

$$\frac{|\dot{H}|}{H^2} \ll 1 \tag{39}$$

This condition is automatically satisfied in exact de Sitter spacetime, where $\dot{H} = 0$.

During radiation domination and matter domination, however, the Hubble parameter decreases rapidly. In the standard cosmological evolution, one has $|\dot{H}|/H^2 = 2$ during radiation domination and $|\dot{H}|/H^2 = 3/2$ during matter domination. Neither regime is therefore adiabatic according to Eq. (39).

This is not necessarily problematic for QCD-DE, since dark energy is observationally required to be subdominant at early times. The more interesting question is how the topological contribution becomes important as the Universe evolves away from matter domination and approaches a de Sitter-like state.

## 5.2 An analytical interpolation between matter domination and de Sitter expansion

To investigate this transition, Van Waerbeke and Zhitnitsky (2025) considered a generalisation of the asymptotic expression

$$\rho_{\rm DE} = c_{\bar{H}} \bar{H} \Lambda_{\rm QCD}^3$$

to a time-dependent form,

$$\rho_{\rm DE} = c_H H \Lambda_{\rm QCD}^3.$$

Here the Hubble parameter $H$ is allowed to evolve with time, while $c_H$ is treated as a constant coefficient. The purpose of this approximation is to construct a cosmological solution that interpolates between an early matter- and radiation-dominated Universe and an asymptotic de Sitter state, and then examine whether the resulting evolution satisfies the adiabatic condition.

For a spatially flat Universe containing matter, radiation, and this QCD-DE contribution, the Friedmann equation is

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_{\rm DE}).$$

Matter and radiation satisfy the usual scaling relations, $\rho_m \propto a^{-3}$ and $\rho_r \propto a^{-4}$.

Introducing an arbitrary reference epoch $a_i$ in the past, at which DE is negligible, and denoting the corresponding Hubble parameter and density parameters by $H_i$, $\Omega_{m,i}$, and $\Omega_{r,i}$, the Friedmann equation becomes a quadratic equation for $H$:

$$H^2 - \bar{H}H - H_i^2\left[\Omega_{m,i}\left(\frac{a_i}{a}\right)^3 + \Omega_{r,i}\left(\frac{a_i}{a}\right)^4\right] = 0.$$

The coefficient $\bar{H}$ is the asymptotic Hubble rate, obtained when matter and radiation become negligible. It is related to the QCD parameters through

$$\bar{H} = \frac{8\pi G}{3} c_H \Lambda_{\rm QCD}^3.$$

The quadratic equation admits two branches. Selecting the expanding solution gives

$$H(a) = \frac{\bar{H}}{2}\left[1 + \sqrt{1 + B\left(\frac{a_i}{a}\right)^3 + C\left(\frac{a_i}{a}\right)^4}\right], \tag{40}$$

where the dimensionless coefficients are

$$B \equiv 4\left(\frac{H_i}{\bar{H}}\right)^2 \Omega_{m,i}, \qquad C \equiv 4\left(\frac{H_i}{\bar{H}}\right)^2 \Omega_{r,i}.$$

This solution has two useful limits. At early times, when $a \ll a_i$ and matter or radiation dominates, the terms inside the square root are large. The standard expansion laws are then approximately recovered: $H \propto a^{-3/2}$ during matter domination and $H \propto a^{-2}$ during radiation domination.

At late times, as $a \to \infty$, the matter and radiation contributions vanish, and the solution approaches

$$H(a) \longrightarrow \bar{H}.$$

Thus the expansion evolves naturally towards a de Sitter attractor.

### 5.3 Is the present Universe adiabatic?

Having obtained an explicit expression for $H(a)$, we can evaluate its time derivative using

$$\dot{H} = \frac{dH}{da}\dot{a} = aH\frac{dH}{da}.$$

Following the derivation in Van Waerbeke and Zhitnitsky (2025), the adiabatic condition can also be expressed in terms of the cosmological energy densities and the DE EoS parameter $w$.

Neglecting radiation at late times, the condition becomes

$$\left(1 + w\frac{\rho_{\rm DE}}{\rho_{\rm DE} + \rho_{\rm M}}\right) \ll \frac{2}{3},$$

where $\rho_{\rm M}$ is the matter density and $w = p_{\rm DE}/\rho_{\rm DE}$.

This expression is useful because it connects the microscopic requirement of adiabatic evolution to quantities familiar from observational cosmology.

At the present epoch, $z = 0$, the estimate is

$$\left(1 + w\frac{\rho_{\rm DE}}{\rho_{\rm DE} + \rho_{\rm M}}\right) \simeq 0.5.$$

The result is smaller than unity, but it is not sufficiently small to satisfy the strong inequality required by the adiabatic approximation. In other words, the Universe is approaching the regime in which the topological contribution may be described adiabatically, but it has not yet reached the asymptotic de Sitter limit.

Consequently, simply replacing $\bar{H}$ by $H(t)$ in the DE expression,

$$\rho_{\rm DE} = c_{\bar{H}} H \Lambda_{\rm QCD}^3,$$

cannot be regarded as a fully justified microscopic calculation of the present-day vacuum energy. It is an interpolation whose range of validity must be examined.

### 5.4 Introducing the switch function $\beta$

To account for the gradual onset of the topological vacuum contribution, Van Waerbeke and Zhitnitsky (2025) introduced a dimensionless *switch function* $\beta(a)$. Its purpose is to describe how the large-scale effects associated with instanton tunnelling become cosmologically relevant as the expansion approaches the adiabatic regime.

The DE density is then parametrised as

$$\rho_{\rm DE} = \beta(a) c_{\bar{H}} H(a) \Lambda_{\rm QCD}^3,$$

with

$$0 \le \beta(a) \le 1.$$

The function is required to approach unity in the asymptotic de Sitter regime, where the topological contribution reaches its full amplitude, while being strongly suppressed at earlier epochs:

$$\begin{cases} \beta(a \to \infty) = 1, \\ \beta(z \gtrsim 1\text{–}2) \sim 0. \end{cases}$$

These conditions encode a simple physical picture. At high redshift, the rapidly evolving cosmological background suppresses the large-scale topological vacuum contribution. As the

expansion slows and the geometry approaches de Sitter spacetime, the contribution progressively becomes active.

In principle, $\beta(a)$ should be derived by calculating the non-perturbative response of the QCD vacuum to an expanding FLRW background. Such a first-principles calculation is not currently available. The switch function is therefore a phenomenological parametrisation motivated by the proposed adiabatic activation mechanism, whose precise redshift dependence remains to be constrained by observations.

### 5.5 Revisiting the Friedmann equation

With the switch function included, the Friedmann equation remains quadratic in the Hubble parameter, but the coefficient of the term linear in $H$ now evolves with time.

It is convenient to introduce the dimensionless time coordinate

$$\tau \equiv \frac{8\pi G}{3}\Lambda_{\rm QCD}^3 c_H\, t.$$

Since $\dot{a} = aH$, the resulting evolution equation for the scale factor becomes

$$\frac{da}{d\tau} = \beta(\tau)\frac{a}{2}\left[1 + \sqrt{1 + \frac{B}{\beta^2(\tau)}\left(\frac{a_i}{a}\right)^3 + \frac{C}{\beta^2(\tau)}\left(\frac{a_i}{a}\right)^4}\right], \tag{41}$$

where $B$ and $C$ retain their previous definitions.

This equation describes the background expansion in the presence of a time-dependent QCD-DE contribution, with two limiting regimes:

- Late times ($\beta \to 1$): The solution reduces to the analytical interpolation derived earlier and asymptotically approaches de Sitter expansion.
- Early times ($\beta \to 0$): The dark-energy contribution is suppressed, recovering the conventional matter- and radiation-dominated expansion.

The function $\beta(a)$ therefore controls both the epoch at which QCD-DE becomes important and the rate at which its contribution develops. Its parametrisation and comparison with cosmological observations provide the connection between the microscopic QCD-DE proposal and the measurable expansion history of the Universe.

### 5.6 Effective dark-energy equation of state

The switch function $\beta(a)$ determines how the QCD vacuum contribution evolves with cosmic time. To compare this prediction with conventional DE models and cosmological observations, it is useful to express the resulting expansion history in terms of an effective EoS parameter $w(z)$.

In standard cosmology, the equation of state relates the pressure and energy density of a dark-energy component through

$$P_{\rm DE} = w\rho_{\rm DE}. \tag{42}$$

In QCD-DE, however, dark energy does not arise from a new dynamical fluid or scalar field. The parameter $w$ is therefore an effective description of the gravitational influence of the evolving QCD vacuum, rather than an independent microscopic degree of freedom.

Once a switch function $\beta(a)$ is specified, Eq. (41) determines the expansion history $H(a)$. Combining the Friedmann and acceleration equations then allows us to reconstruct the effective equation of state. Following Van Waerbeke and Zhitnitsky (2025), one obtains

$$1 + w = \frac{-2\dot{H} - 3H_i^2\Omega_{m,i}\left(\frac{a_i}{a}\right)^3 - 4H_i^2\Omega_{r,i}\left(\frac{a_i}{a}\right)^4}{3c_H\bar{H}H}. \tag{43}$$

This relation illustrates an important feature of the model: the effective EoS is not prescribed independently, but follows from the cosmological evolution generated by the switch function.

As discussed in Van Waerbeke and Zhitnitsky (2025), the QCD-DE framework does not impose a fixed functional form on $w(z)$. Depending on the evolution of $\beta(a)$, the effective equation of state may depart from $w = -1$ and potentially cross the phantom divide. This distinguishes the approach from the Chevallier–Polarski–Linder parametrisation (Chevallier and Polarski, 2001), which assumes a specific redshift dependence of $w(z)$ without specifying its microscopic origin. The QCD-DE approach instead parametrises the activation of a proposed vacuum mechanism, from which the effective $w(z)$ is derived. Nevertheless, because the switch function is not yet determined from first principles, its observational reconstruction remains essential for testing the model.

## 6 Cosmological constraints and comparison with ΛCDM and CPL

Lee et al. (2026) performed the first comprehensive cosmological analysis of QCD-DE, investigating whether the proposed QCD vacuum contribution can reproduce the observed expansion history of the Universe. The analysis combines CMB measurements from Planck, ACT, and SPT-3G, BAO from DESI DR2, and Type Ia supernova distances from Pantheon+ and DES-Dovekie. Cosmological parameters were constrained using Markov-chain Monte Carlo sampling, while model comparisons employed goodness-of-fit statistics, the Akaike and Deviance Information Criteria, and Bayesian evidence.

The principal findings are:

- **Expansion history.** QCD-DE reproduces the late-time expansion history favoured by the combined observations, achieving a goodness of fit comparable to CPL. Both evolving-DE descriptions can improve the fit relative to ΛCDM, particularly when DESI BAO measurements are combined with supernova distances. This demonstrates that the QCD-DE framework has sufficient flexibility to reproduce the deviations from a cosmological constant suggested by current data.

- **Phantom-divide crossing.** The reconstructed effective equation of state evolves across $w = -1$ near $z \simeq 0.67$. Such behaviour is difficult to realise with a conventional, minimally coupled canonical scalar field.

- **Statistical model comparison.** Although evolving-DE models introduce additional parameters relative to ΛCDM, the improvement in fit can compensate for the associated complexity penalty. For certain combinations of CMB, BAO, and supernova data, the Bayesian evidence moderately favours QCD-DE over ΛCDM. Its statistical support also appears more stable than that of CPL when changing between the Pantheon+ and DES-Dovekie supernova compilations. However, the preference depends on the datasets and remains insufficient to establish a decisive detection of evolving dark energy.

These results establish QCD-DE as an observationally competitive description of dark energy, comparable in performance to CPL while offering a proposed microscopic origin for its evolution. Nevertheless, the switch function remains phenomenological, and the observational evidence is not yet conclusive. A first-principles derivation of $\beta(a)$, together with independent observational tests, will be essential for determining whether QCD-DE provides a genuine physical explanation of cosmic acceleration.

## 7 Revisiting the three dark energy challenges

The QCD-DE framework developed by Van Waerbeke and Zhitnitsky (2025) offers a possible common explanation for the three fundamental challenges of dark energy.

- **Challenge 1: The small amplitude.** The proposed Minkowski-space subtraction removes the enormous QCD vacuum contribution proportional to $\Lambda_{\rm QCD}^4$, leaving a residual energy density $\rho_{\rm DE} \sim c_{\bar{H}} \bar{H} \Lambda_{\rm QCD}^3$. Combining this expression with the Friedmann equation produces an energy density of the observed order of magnitude, using QCD and gravitational scales. This is a significant theoretical result, but the subtraction prescription and linear Hubble correction remain to be established from first principles in four-dimensional cosmology.

- **Challenge 2: Late-time activation.** The proposed mechanism associates the emergence of dark energy with the increasingly adiabatic evolution of the expanding Universe. Topological vacuum contributions are suppressed during earlier epochs and become significant as the expansion approaches the de Sitter regime. This provides a physical motivation for late-time activation, represented by the switch function $\beta(a)$. However, its precise evolution is not yet derived microscopically and must be constrained observationally.

- **Challenge 3: Evolving dark energy.** The time-dependent activation of the QCD vacuum naturally produces an effective equation of state $w(z)$ that need not remain equal to $-1$ and may cross the phantom divide. The framework can accommodate the evolving dark-energy behaviour suggested by recent DESI data without introducing a new fundamental dark-energy field. Nevertheless, these observational indications remain unconfirmed, and the flexibility of $\beta(a)$ limits the model's present predictive power.

## 8 Conclusion

The QCD-DE model offers a physically motivated explanation for cosmic acceleration rooted in the non-perturbative topological structure of the QCD vacuum, without introducing new fundamental fields or energy scales. Unlike the phenomenological CPL parametrization, which describes the possible evolution of dark energy without explaining its physical origin, QCD-DE connects cosmic acceleration to a known sector of the Standard Model. Remarkably, the QCD scale naturally generates a vacuum energy density and cosmological expansion timescale of the observed order of magnitude.

On the observational front, QCD-DE reproduces the evolving dark energy behaviour suggested by DESI, achieving a goodness of fit comparable to CPL. Bayesian model comparison moderately favours QCD-DE over ΛCDM for certain combinations of CMB, BAO and supernova data, while individual datasets remain consistent with ΛCDM.

Although the underlying cosmological vacuum-energy mechanism still requires further theoretical development, QCD-DE provides a promising framework connecting fundamental particle physics to cosmic acceleration, with distinctive predictions that can be tested by future cosmological observations.

## References

Y. Aharonov and D. Bohm. Significance of electromagnetic potentials in the quantum theory. *Phys. Rev.*, 115:485–491, Aug 1959. doi:10.1103/PhysRev.115.485. URL https://link.aps.org/doi/10.1103/PhysRev.115.485.

Y. Aharonov and A. Casher. Topological quantum effects for neutral particles. *Phys. Rev. Lett.*, 53:319–321, Jul 1984. doi:10.1103/PhysRevLett.53.319. URL https://link.aps.org/doi/10.1103/PhysRevLett.53.319.

Michel Chevallier and David Polarski. Accelerating Universes with Scaling Dark Matter. *International Journal of Modern Physics D*, 10(2):213–223, January 2001. doi:10.1142/S0218271801000822.

A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn. Baryon structure in the bag theory. *PRD*, 10 (8):2599–2604, October 1974a. doi:10.1103/PhysRevD.10.2599.

A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf. New extended model of hadrons. *PRD*, 9(12):3471–3495, June 1974b. doi:10.1103/PhysRevD.9.3471.

A. Cimmino, G. I. Opat, A. G. Klein, H. Kaiser, S. A. Werner, M. Arif, and R. Clothier. Observation of the topological aharonov-casher phase shift by neutron interferometry. *Phys. Rev. Lett.*, 63:380–383, Jul 1989. doi:10.1103/PhysRevLett.63.380. URL https://link.aps.org/doi/10.1103/PhysRevLett.63.380.

Sidney Coleman. More about the massive schwinger model. *Annals of Physics*, 101(1):239–267, 1976. ISSN 0003-4916. doi:https://doi.org/10.1016/0003-4916(76)90280-3. URL https://www.sciencedirect.com/science/article/pii/0003491676902803.

Dong Ha Lee, Carsten van de Bruck, Eleonora Di Valentino, Ludovic Van Waerbeke, and Ariel Zhitnitsky. Evolving Dark Energy Is Vacuum Energy After All. *arXiv e-prints*, art. arXiv:2606.20036, June 2026. doi:10.48550/arXiv.2606.20036.

Günter Plunien, Berndt Müller, and Walter Greiner. The Casimir effect. *Physics Reports*, 134 (2-3):87–193, March 1986. doi:10.1016/0370-1573(86)90020-7.

Julian Schwinger. Gauge invariance and mass. ii. *Phys. Rev.*, 128:2425–2429, Dec 1962. doi:10.1103/PhysRev.128.2425. URL https://link.aps.org/doi/10.1103/PhysRev.128.2425.

I. L. Shapiro and J. Solà. Scaling behavior of the cosmological constant and the possible existence of new forces and new light degrees of freedom. *Physics Letters B*, 475(3-4):236–246, March 2000. doi:10.1016/S0370-2693(00)00090-3.

Ilya L. Shapiro and Joan Solà. The scaling evolution of the cosmological constant. *Journal of High Energy Physics*, 2002(2):006, February 2002. doi:10.1088/1126-6708/2002/02/006.

Akira Tonomura, Nobuyuki Osakabe, Tsuyoshi Matsuda, Takeshi Kawasaki, Junji Endo, Shinichiro Yano, and Hiroji Yamada. Evidence for aharonov-bohm effect with magnetic field completely shielded from electron wave. *Phys. Rev. Lett.*, 56:792–795, Feb 1986. doi:10.1103/PhysRevLett.56.792. URL https://link.aps.org/doi/10.1103/PhysRevLett.56.792.

Federico R. Urban and Ariel R. Zhitnitsky. Cosmological constant from the ghost: A toy model. *PRD*, 80(6):063001, September 2009. doi:10.1103/PhysRevD.80.063001.

Federico R. Urban and Ariel R. Zhitnitsky. The QCD nature of dark energy. *Nuclear Physics B*, 835(1-2):135–173, August 2010a. doi:10.1016/j.nuclphysb.2010.04.001.

Federico R. Urban and Ariel R. Zhitnitsky. The cosmological constant from the QCD Veneziano ghost. *Physics Letters B*, 688(1):9–12, April 2010b. doi:10.1016/j.physletb.2010.03.080.

Ludovic Van Waerbeke and Ariel Zhitnitsky. DESI results and Dark Energy from QCD topological sectors. *arXiv e-prints*, art. arXiv:2506.14182, June 2025. doi:10.48550/arXiv.2506.14182.

Ariel R. Zhitnitsky. QCD as a topologically ordered system. *Annals of Physics*, 336:462–481, September 2013. doi:10.1016/j.aop.2013.05.020.

Ariel R. Zhitnitsky. Dynamical de Sitter phase and nontrivial holonomy in strongly coupled gauge theories in an expanding universe. *PRD*, 92(4):043512, August 2015. doi:10.1103/PhysRevD.92.043512.